Electron Current Collection by a Bare Tether in Mesothermal Conditions

Tatsuo Onishi* and Manuel Martínez-Sánchez†

Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

David L. Cooke
Air Force Research Laboratory
Space Vehicle Directorate, Hanscom AFB, MA, USA

(*)email: onishi@mit.edu, tel: 1-617-253-7485, fax: 1-617-258-5940

Abstract

In tethered satellite technology, it is important to estimate how many electrons a spacecraft can collect from its ambient plasma by its bare electrodynamic tether. The analysis is however very difficult because of the small but significant Geomagnetic field and the spacecraft’s relative motion to both ions and electrons. The object of the work reported here is the development of a numerical method, for this purpose. Particle-In-Cell (PIC) method, for the calculation of electron current to a positive bare tether moving at orbital velocity in the ionosphere, i.e. in a flowing magnetized plasma under Maxwellian collisionless conditions. In a PIC code, a number of particles are distributed in phase space and the computational domain has a grid on which Poisson equation is solved for field quantities. The code uses the quasi-neutrality condition to solve for the local potential at points in the plasma which coincide with the computational outside boundary. The quasi-neutrality condition imposes \( n_e = n_i \) on the boundary, electron density, \( n_e \), and ion density, \( n_i \), have both incoming and outgoing parts. Outgoing particle densities are calculated numerically, and incoming particle densities are calculated analytically. In this calculation, electrons are assumed to have a shifted Maxwellian distribution at the boundary. The Poisson equation is solved in such a way that the presheath region can be captured in the computation. Results show that the collected current is higher than the Orbital Motion Limit (OML) theory. The OML current is derived under the steady isotropic Maxwellian condition by calculating all particles whose trajectories can be traced back to infinity from the surface of a collector [10]. Therefore we know that the distribution function of electrons on the collector’s surface is Maxwellian except that particles corresponding to negative total energy are excluded. This gives rise to the upper limit of current collection in a steady state. However the prediction may not be straightforward as seen in the space experiment using a spherical collector (TSS-1R), where a different “upper bound” was seen to be broken.

Introduction

Sanmartín, Martínez-Sánchez and Ahedo [1] proposed a thin bare electrodynamic tether to collect currents in the Orbital-Motion-Limit (OML) regime. The OML current is derived under the steady isotropic Maxwellian condition by calculating all particles whose trajectories can be traced back to infinity from the surface of a collector [10]. Therefore we know that the distribution function of electrons on the collector’s surface is Maxwellian except that particles corresponding to negative total energy are excluded. This gives rise to the upper limit of current collection in a steady state. However the prediction may not be straightforward as seen in the space experiment using a spherical collector (TSS-1R), where a different “upper bound” was seen to be broken.

TSS-1R space experiment which took place in 1996 brought about unexpected results. The experiment used a spherical collector, whose radius is much larger than the Debye length of unperturbed plasma. Previously it was expected that the current collection would have an upper limit which was derived from the canonical angular momentum conservation–Parker-Murphy model [2]. The result was that TSS-1R collected more current than the Parker-Murphy predictions. An electron temperature increase in the near presheath was also observed. In order to explain the current enhancement, Cooke and Katz used a fluid model to relate the potential increase to the temperature increase, assuming that there are trapped electrons in the presheath, being trapped for long enough to use the fluid approximation [3]. Laframboise introduced the concept of magnetic presheath to modify (enlarge) the Parker-Murphy collection “tube” [6]. These theories still await experimental and/or
computational results for its verification. For a thin tether, the Parker-Murphy limit may be superseded by the OML limit as the upper bound of collected current. However, some of the same phenomena discussed in [3] and [6] may be still involved.

In order to predict current collection to a bare electrodynamic tether, we have developed a numerical code using Particle-In-Cell (PIC) method. The PIC method has been established to analyze collisionless plasmas. Especially it works well to simulate the particle-field interaction [11]. We incorporate the quasi-neutrality condition at the boundary developed by the authors, to capture the presheath region. This scheme is shown to give good quantitative approximations in the prediction of current collection to a cylindrical probe in and out of the OML regime, in the case of a quiescent unmagnetized plasma [7]. In the ionosphere, at orbital speed where the bare tether will be put in practice, so-called “mesothermal condition” applies. Mesothermal condition is that the tether’s orbital velocity, \( U_{tether} \), is much faster than the ion thermal speed, \( v_{t,i} \), and much slower than the electron thermal speed, \( v_{t,e} \).

\[
v_{t,i} \ll U_{tether} \ll v_{t,e} \tag{1}
\]

In this work, we presents recent results from our PIC computations and discuss the effects of the mesothermal condition on the current collection to a bare tether. We are in a process of development of a new larger grid system to reduce the boundary effects.

**Mesothermal Condition**

In our computation we assume that ions are Maxwellian at the far upstream region, and approaching the tether one-dimensionally at the tether’s orbital speed. Electrons are assumed to have shifted Maxwellian distribution functions at the computational outside boundaries.

\[
f_e(w) = A_e \exp \left( - \frac{m_e(w_x - U_{tether})^2 + w_y^2 + w_z^2}{2\kappa T_e} \right) \tag{2}
\]

where \( A_e = n_\infty m_e/(2\pi\kappa T_e)^{3/2} \exp(-q\phi/\kappa T_e) \), \( n_\infty \) electron density at infinity, \( m_e \) electron mass, \( T_e \) electron temperature, \( q \) electric charge of electron, \( \phi \) local potential, \( \kappa \) Boltzmann constant.

A mesothermal condition defined by equation (1) and the very high positive potential on the tether may induce instability. It may be explained in terms of Bohm stability criterion. The Bohm sheath stability criterion claims that throughout the shielding region surrounding a probe the sign of the charge density must be opposite that of the probe, and otherwise the Bohm unstable sheath suffers fluctuations. In order to satisfy the Bohm criterion, electron density needs to be more than that given by Maxwellian.

### Computation

The major difficulty of a PIC method applied to an infinitely large plasma appears in the specification of the computational outside boundary condition, namely the velocity distribution function at a boundary point. In order to treat the boundary, we assume a shifted Maxwellian velocity distribution for electrons at the boundary and a shifted Maxwellian velocity function for ions at the far upstream. Based on the assumption that ions are Maxwellian in the far upstream region and are accelerated/decelerated one-dimensionally, we obtain the ion distribution function at the boundary as a function of the local potential. From this function, we calculate the density, which is required for the quasi-neutrality condition, and the flux, which is required to calculate the number of ions to be replenished into the computational domain per timestep [8]. The density and flux of incoming electrons are also calculated, based on the assumption that electrons have a shifted Maxwellian distribution at the boundary [8].

To avoid an error associated with a very large velocity of electrons near the high potential tether, sub-iterations are used for fast electrons. Tether charge is also kept at \( 7eV \), which is above the ion ram energy, reducing the number of very fast particles near the tether. Higher potential case is possible without any practical problems. We are in the process of updating a grid system from cylindrical grids to square grids, to be able to expand the computational region without inducing numerical instabilities. The expansion of structured cylindrical grids inevitably increases a cell size at the outer edge. A large cell size is known to be numerically unsta-

**Table 1: Plasma parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field</td>
<td>0.3 Gauss</td>
</tr>
<tr>
<td>Ion mass (O+)</td>
<td>( 2.67 \times 10^{-26} ) kg</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>0.1 eV</td>
</tr>
<tr>
<td>Ion temperature</td>
<td>0.1 eV</td>
</tr>
<tr>
<td>Electron thermal velocity</td>
<td>212 km/sec</td>
</tr>
<tr>
<td>Ion thermal velocity</td>
<td>1 km/sec</td>
</tr>
<tr>
<td>Satellite speed</td>
<td>8 km/sec</td>
</tr>
<tr>
<td>Electron gyro radius</td>
<td>2.5 cm</td>
</tr>
<tr>
<td>Ion gyro radius</td>
<td>430 cm</td>
</tr>
<tr>
<td>Electron (Ion) density</td>
<td>( 10^{11}/m^3 )</td>
</tr>
<tr>
<td>Debye length at infinity</td>
<td>0.74 cm</td>
</tr>
</tbody>
</table>


ble [11] In this paper, results from simulations performed on both grids will be presented.

We have been using the cylindrical grid system for simulations of different plasma conditions, namely quiescent unmagnetized case, quiescent magnetized case, flowing unmagnetized case and flowing magnetized case. In each case, current collection was calculated for the Debye-to-radius ratio equal to one. Results are shown in Table 2. Numbers are normalized by the OML current. In both quiescent cases, current collection is very close to the OML, whereas in flowing cases, the tether collects about 2 times higher than the OML current.

In Figures 1 and 2, electron density, ion density, electric charge density and potential field are shown for both magnetized and unmagnetized cases. In the ion density map, it is clearly seen that a wake is formed behind the tether due to the mesothermal condition. In order to keep the quasi-neutrality most electrons are excluded from this wake region, where potential becomes negative. Only a few very energetic electrons can penetrate this region. In the immediate vicinity of the tether, electrons shield the tether and create a sheath region. The difference between these two cases may be seen in the potential profiles. in the magnetized case, electron motions are restricted along the Geomagnetic field, which runs vertically in the map, and thus magnetic wings are extending to the both sides of the tether, although they appear to be limited by the computational boundaries.

From the viewpoint of the quasi-neutrality, it is reasonable for electrons to have higher density in the vicinity of the tether, where ion density is also elevated due to the ion’s quasi one-dimensional motion and the high potential on the tether. However from the calculation of steady state collisionless Maxwellian distribution, especially in the unmagnetized case, electron density is at most equal to that at infinity, or rather less than that because of the electron collection on the tether. Laframboise and Parker [10] explained that in 2-D steady isotropic Maxwellian condition, electron density is obtained by considering all the particles with its total energy, $E > 0$.

$$n_e = \int_{w^2+w_y^2+\frac{2eE}{m_e}}^{\infty} A_w \exp \left( - \frac{m_e(w_x^2+w_y^2)}{2\kappa T_e} \right) \, dw$$

$$w^2+w_y^2 > \frac{2eE}{m_e}$$

$$n_\infty$$

(4)

Even though we are missing a flowing effect in this calculation, the mesothermal condition that electron thermal velocity is much larger than the tether orbital speed, makes the effect negligible. If the electron density is equal to or less than that at infinity, the Bohm stability criterion is violated, and thus the plasma becomes unstable. The instability attracts electrons in order to satisfy the Bohm stability criterion, and plasma becomes marginally stable. The mechanism of attracting more electrons is not clear at this moment. However as seen in Figure 3, we may partly attribute it to the particle trapping due to the field oscillation. The figure shows several particle trajectories observed in a computation. Some electrons are trapped in the ram side of the tether, wander around and eventually get collected by the tether. Trapped electrons mostly oscillates up-and-down in Figure 3. In part (c) of Figure 4, there are more particles with such velocities than those with other component of velocity, whereas in part (a) the distribution is more uniform with respect to the velocity component except for the open part which corresponds to particles collected by the tether. This means that once an electron is trapped, it is pulled by the tether potential and stays trapped near the tether.

We also see the effects of outside boundaries. Potentials tend to be lower by at most 0.1 eV immediately inside the boundary. This potential drop is clearly seen in the computation which uses square grids (Figure 5). A contour of equipotential goes parallel with the outside computational boundary. Once this potential drop appears near the boundary, the boundary behaves like a negatively charged wall and maintains a positive charge sheath next to itself. This potential drop is also seen in the cylindrical grids at least on some spots. The “sheath” along the outside boundary observed in the square grids is stable and continuous because the grid size is much larger than the cylindrical one and thus does not get disturbed from other parts of boundary.

To see how this potential drop changes current collection, let us look at Figure 4 again. We don’t see many particles with a small total energy. As a preliminary result, we may expect that the potential drop at the boundary does not change the current collection considerably.
Conclusion and Future Work

A Particle-In-Cell method has been developed for the calculation of current collection by a moving bare tether. Current collection enhancement has been recognized in the computation. As the Bohm stability criterion claims, electron density becomes higher than Maxwellian where ion density increases one-dimensionally in the ram front of a tether. The understanding of electron density increase is still incomplete, however, we have detected particle trap-pings occurring in the ram region, and corresponding particle-field interactions. We also identified the computational outside boundary effects. There are potential drops in the immediate vicinity of the boundary which behaves like a negatively charged wall.

The preliminary examination of distribution function suggests that the potential drop at the boundary may not induce a considerable change in the current collection. It is also observed that electrons are trapped mostly in the immediate vicinity of the tether.

Works to be done in the near future includes the elimination of the potential drop near the outside boundary, to obtain correct local potential, which is important to calculate the number of injected particles, and the modification of boundary conditions in order to prohibit the reflection of wave radiations at the boundary.

Acknowledgments

This work was supported through a grant from the USAF space vehicle directorate (Technical monitor: David L. Cooke).

References


[8] Onishi, T., M. Martinez-Sanchez and D.L. Cooke Computation of Current to a Moving Bare Tether, IEPC99-217, 1999


Figure 1: Flowing unmagnetized plasma: Maps of electron density (upper left), ion density (upper right), electric charge density (lower left) and potential field (lower right). Densities are normalized by that at infinity and potential by the equivalence of plasma energy ($kT_e=0.1\text{eV}$). Tether potential is kept at 250 (25eV).

Figure 2: Flowing magnetized plasma: Maps of electron density (upper left), ion density (upper right), electric charge density (lower left) and potential field (lower right). Densities are normalized by that at infinity and potential by the equivalence of plasma energy ($kT_e=0.1\text{eV}$). Tether potential is kept at 250 (25eV).
Figure 3: Electron trajectories

(a) Particle Trapping  
(b) Particle Trapping  
(c) Boundary Effect

Figure 4: Time averaged distribution functions measured at different points ahead of the Tether (in the ram side): White circles corresponds to $\sqrt{2e \phi/m_e}$.

(a) 15 Debye lengths ahead of the Tether : $\phi = 0.67$eV  
(b) 5 Debye lengths ahead of the Tether : $\phi = 2.4$eV  
(c) 2.5 Debye lengths ahead of the Tether : $\phi = 4.15$eV

Figure 5: Flowing unmagnetized plasma on square grids: Maps of electron density (upper left), ion density (upper right), electric charge density (lower left) and potential field (lower right). Densities are normalized by that at infinity and potential by the equivalence of plasma energy ($kT_e=0.1$eV). Tether potential is kept at 70 (7eV). The equipotential contour just inside the outside boundary corresponds to 1 (0.1eV).